

Quasidensity, type (FP) and type (FPV)

Stephen Simons *

Abstract

Let E be a nonzero real Banach space.

Quasidensity is a concept that can be applied to subsets of $E \times E^*$ (or equivalently to multifunctions from E into E^*).

Every closed quasidense monotone set is *maximally monotone*, but there exist maximally monotone sets that are not quasidense.

The subdifferential of a proper, convex lower semicontinuous function on E is quasidense. The subdifferentials of certain *nonconvex* functions are also quasidense. (This follows from joint work with Xianfu Wang.)

The closed monotone quasidense sets satisfy a *sum theorem* and a *dual sum theorem*.

We know of at least eight conditions equivalent to the statement that a closed monotone set be quasidense, but quasidensity seems to be the only one that extends easily to *nonmonotone* sets.

We also discuss multifunctions of type (FPV) = *maximal monotone locally* and type (FP) = *locally maximal monotone*.

*University of California, Santa Barbara, stesim38@gmail.com