Descent modulus and abstract determination of functions

David Salas *

Abstract

In 2018, Boulmezaoud, Cieutat and Daniilidis [1] obtained a very unexpected determination result: For any two convex functions f, g over a Hilbert space $\mathcal{H}$, that are of class $\mathcal{C}^2$ and bounded from below, one has that

$$(\|\nabla f(x)\|= \|\nabla g(x)\|, \quad \forall x \in \mathcal{H}) \implies f = g \text{ up to a constant.}$$

This implication is what we call a determination result. In this presentation we will quickly survey the advances emanating from this seminal result, providing extensions in the convex case in vector spaces [4, 5], and extensions to nonconvex settings in metric spaces [3, 5]. Then, we will present an abstract setting of operators enjoying the determination result, which we call *Descent Modulus*. In a nutshell, a Descent Modulus is an abstract way to measure the *descent of a function at a given point*. We will show how to use a Descent Modulus to determine functions and what kind of operators are descent modulus. Finally, we will visit descent modulus in finite spaces, showing that any "reasonable" descent modulus can be described as a slope-like operator over a system of neighborhoods. At the end of the presentation, we will discuss some open questions and perspectives of this line of research.

The work presented here has been done in collaboration with A. Daniilidis, T. M. Le and L. Miclo.

References

[1] T. BOULMEZAOUD, P. CIEUTAT, A. DANIILIDIS, Gradient flows, second-order gradient systems and convexity, *SIAM J. Optim.* **28** (2018), 2049–2066.

[2] A. DANIILIDIS, L. MICLO AND D. SALAS. Descent Modulus and Applications. *Preprint arXiv:2211.118198*, 2022.

*Instituto de Ciencias de la Ingeniería, Universidad de O'Higgins, david.salas@uoh.cl

[3] A. DANIILIDIS AND D. SALAS. A determination theorem in terms of the metric slope. *Proc. Amer. Math. Soc.*, **150**(10):4325–4333, 2022.

[4] P. PÉREZ-AROS, D. SALAS, AND E. VILCHES. Determination of convex functions via subgradients of minimal norm. *Math. Program.*, **190**(1-2, Ser. A):561–583, 2021.

[5] L. THIBAULT AND D. ZAGRODNY. Determining functions by slopes. *Comm. Contemp. Math.*, **25**(7): 250014, 2023.