One-sided linear couplings

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Abstract

In a Euclidean space, the Fenchel coupling by scalar product is linear both in the primal and in the dual variable, variables which here belong to the same space. By contrast, a one-sided linear (OSL) coupling is a function — from the Cartesian product of a set with a Euclidean space towards the real numbers — which is linear only in the dual variable. As such, OSL conjugacies share some properties with the Fenchel conjugacy.

In a first part, we provide, for an OSL coupling and a numerical function defined over the primal set, expressions of the first and second conjugate, of the subdifferential, of *generalized Fenchel-Young and Fitzpatrick gaps* (two notions we introduce), of proximal operator (deduced from suitable Bregman distances). We characterize "convex" functions for an OSL coupling.

In a second part, we present the notions of *convex factorization* and of *convex extension*, and we show how they are related to a subclass of OSL couplings.

In a third part, we review a series of results on a subclass of OSL couplings — so-called Capra (Constant Along Primal Rays) — and how they reveal hidden convexity in the ℓ_0 pseudonorm and, more generally, in 0-homogeneous functions. We also discuss Capra-polarities.

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References

[1] J.-P. Chancelier and M. De Lara. Hidden convexity in the l_0 pseudonorm. *Journal of Convex Analysis*, 28(1):203–236, 2021.

[2] J.-P. Chancelier and M. De Lara. Capra-convexity, convex factorization and variational formulations for the l_0 pseudonorm. *Set-Valued and Variational Analysis*, 30:597–619, 2022.

[3] J.-P. Chancelier and M. De Lara. Constant along primal rays conjugacies and the l_0 pseudonorm. *Optimization*, 71(2):355–386, 2022.

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[4] J.-P. Chancelier and M. De Lara. Orthant-strictly monotonic norms, generalized top-k and k-support norms and the l0 pseudonorm. *Journal of Convex Analysis (to appear)*, 2022.